Flux tubes, visons, and vortices in spin–charge separated superconductors

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Abstract

The idea of spin–charge separation in cuprate superconductors has been recently energized by Senthil and Fisher who formulated a Z_2 gauge theory and, within its context, proposed a ''vison detection'' experiment as a test for topological order in a sample with multiply connected geometry. Here we show that the same experiment can be performed to test for the spin–charge separation in U(1) (but not in SU(2)) theory and argue that vortex core spectroscopy can in fact distinguish between the different symmetries of the fictitious gauge field. © 2001 Elsevier Science B.V. All rights reserved.

While there exist number of compelling reasons to think that electron could be fractionalized in high-T_c cuprate superconductors [1], there is thus far no *direct* experimental evidence for this effect in dimensions D higher than 1. Recently, Senthil and Fisher (SF) proposed an experiment [2] to directly test for electron fractionalization in $D = 2, 3$ in strongly underdoped cuprate samples with multiply connected geometries. SF framed their proposal in the context of the Z_2 gauge theory [3]. The purpose of this note is to point out that SF ex-periment constitutes a general test for spin–charge separation, and, as long as the charge carrying boson (be it holon or chargon) condenses without pairing, the outcome will be independent of the symmetry of the fictitious gauge field present in the theory. Thus, if the spin–charge separation takes place in cuprates, the outcome of the SF experiment will be the same for U(1) and Z_2 theories. As pointed out by Lee and Wen [4], it will not work in the SU(2) theory. We furthermore point out that SF experiment is closely tied to the ongoing debate on the structure of a magnetic vortex in spin–charge separated superconductors [4–8] and propose that probing the excitation spectrum in the core of a vortex as a function of doping can shed light on the symmetry of the fictitious gauge field.

The essence of the SF experiment [2] lies in the realization that a singly quantized vortex carrying $hc/2e$ magnetic flux is a very peculiar object in the

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fractionalized superconductor. This is because charge carrying boson of the theory (holon or chargon) is assumed to Bose-condense individually, i.e. without pairing. Such charge-e condensate would normally quantize the magnetic flux in multiples of hc/e. Ordinary superconducting $hc/2e$ vortex would cause the condensate wavefunction to acquire a phase π on encircling the vortex, producing a branch cut. Branch cut in the macroscopic wavefunction would lead to various catastrophic consequences (such as infinite currents) and must be therefore compensated by a feature in the gauge field. In the Z_2 theory this is done by binding "vison", a topological excitation of the Z_2 gauge field, to the singly quantized vortex. Vison supplies the missing π phase to the chargon condensate wavefunction but does not alter the spinon condensate wavefunction because the latter are assumed to condense in singlet pairs and therefore the net phase acquired by paired spinons is 2π.

SF envision trapping a singly quantized vortex in the hole fabricated in a strongly underdoped superconductor. Such hole would then necessarily be threaded by a vison. When heated above the superconducting T_c, the magnetic flux can easily escape from the hole but vison remains trapped because it is a gapped excitation below the pseudogap temperature T^*. Although there is no direct way to detect vison, one could deduce its presence by cooling in zero field back to the superconducting state, where the vison will bind the magnetic flux which can be directly measured.

We now argue that the same will happen in the U(1) theory, the main difference being that the vison will be replaced by a flux quantum [2] of the U(1) gauge field. To illustrate our arguments it is easiest to consider the effective Ginzburg–Landau (GL) theory for fractionalized superconductor formulated originally by Sachdev [5] and by Nagaosa and Lee [6]. The corresponding free energy reads

$$f_{GL} = |(\nabla - 2\mathrm{i}\mathbf{a})\Delta|^2 + r_\Delta |\Delta|^2 + \frac{1}{2}u_\Delta |\Delta|^4$$
$$+ |(\nabla - \mathrm{i}\mathbf{a} - \mathrm{i}e\mathbf{A})b|^2 + r_b |b|^2 + \frac{1}{2}u_b |b|^4$$
$$+ v|\Delta|^2 |b|^2 + \frac{1}{8\pi}(\nabla \times \mathbf{A})^2 + f_{gauge}[\mathbf{a}], \qquad (1)$$

where Δ and b are spinon pair and holon condensate fields respectively, minimally coupled to the electromagnetic field $\mathbf{A}$ and a fictitious U(1) gauge field $\mathbf{a}$.

$$f_{gauge} = \frac{\sigma}{2}(\nabla \times \mathbf{a})^2 \qquad (2)$$

is the gauge field stiffness term which originates from integrating out the high-energy microscopic degrees of freedom. It has been argued recently [9] that σ is small or even zero in realistic models of cuprates. This results in the above GL theory being extreme type-I with respect to $\mathbf{a}$ but extreme type-II with respect to $\mathbf{A}$. In the absence of field the parameters r, u, and v can be chosen such that this GL theory reproduces the standard phase diagram of cuprates shown in Fig. 1. Also, we note that $\mathbf{a}$ is by construction a compact lattice gauge field. Maxwell form of Eq. (2) is a simplified continuum representation of the corresponding lattice expression.
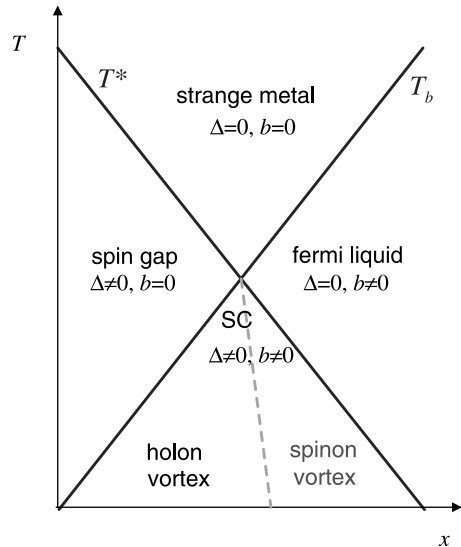

Fig. 1. Schematic phase diagram of a spin–charge separated system within the U(1) theory in the doping-temperature plane, as applied to cuprate superconductors.

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[2] Here and hereafter we reserve the term 'flux quantum' for $hc/2e$, i.e. the conventional superconducting flux quantum, and the corresponding expression $h/2$ for the quantum of fictitious gauge flux.

We first review what is known about the vortex solutions of Eq. (1) and then we discuss SF experiment in the context of U(1) theory. As shown in Refs. [6,7], in the limit of negligible σ the free energy (1) allows for two types of singly quantized vortices: (i) spinon vortex in the overdoped region and (ii) holon vortex in the underdoped region [cf. Fig. 1]. In the spinon vortex the gauge field **a** develops a net flux such that $\mathbf{a} \approx -e\mathbf{A}$ and the singularity occurs in the spinon field [first term in Eq. (1)] with Δ vanishing at the vortex center. In the holon vortex b vanishes in the core and the gauge flux is contracted into a δ-function flux tube at the vortex center. This fictitious flux tube is invisible to the spinon condensate and its main purpose is to heal the branch cut that would otherwise occur in the holon order parameter, just as vison does in the Z_2 theory. In this respect the singly quantized holon vortex in the compact U(1) theory is equivalent to the Z_2 vortex. [3]

The important point to notice is that both types of singly quantized vortices in the U(1) theory bind a flux $h/2$ (in units where magnetic flux quantum is $hc/2e$) of the fictitious gauge field **a**. Thus, when such vortex is trapped in the hole, as envisioned in SF experiment, that hole will be threaded by a single quantum of the fictitious flux. Now consider removing the external field and heating the sample above T_c. The magnetic flux will quickly escape but, in the strongly underdoped material, the fictitious flux will remain trapped below T^*. To see this, consider that according to the standard U(1) phenomenology [10] the mean field state just above T_c in the underdoped region is characterized by $b = 0$ and $|\Delta| > 0$. The free energy (1) therefore becomes

$$f'_{GL} = |(\nabla - 2i\mathbf{a})\Delta|^2 + r_\Delta |\Delta|^2 + \frac{1}{2} u_\Delta |\Delta|^4$$
$$+ \frac{1}{8\pi}(\nabla \times \mathbf{A})^2 + \frac{\sigma}{2}(\nabla \times \mathbf{a})^2. \qquad (3)$$

This expression is formally equivalent to the GL theory for a superconducting order parameter Δ minimally coupled to the U(1) gauge field **a**. The

main point is that below the critical temperature for Δ, which is T^* in this model, free energy Eq. (3) will exhibit the Meissner effect with respect to **a**. On the mean field level the gauge flux threading the hole cannot penetrate into the bulk below T^* and remains trapped there forever. Going beyond the mean-field considerations the fictitious gauge flux can tunnel out of the hole, but sample geometry can be designed in such a way that it remains trapped for sufficiently long times.

The real electromagnetic field **A**, on the other hand, is completely decoupled from the matter fields. This reflects the obvious fact that non-superconducting state above T_c cannot exhibit the true Meissner effect.

We note that the same argument seemingly could be made for the *overdoped* region, where above T_c the mean field state is characterized by $\Delta = 0$ and $|b| > 0$. It would appear that the fictitious flux now remains trapped above T_c by virtue of the Meissner effect caused by the holon condensate. However, upon closer examination one finds that this is not the case because singly condensed holons cannot support $h/2$ flux quantum in the absence of physical magnetic field. Consequently, in the U(1) theory the SF effect will occur in the underdoped but not in the overdoped region, just as in the Z_2 theory.

Having argued that the SF experiment will yield the same outcome irrespective of the symmetry of the gauge field, we now turn to the differences between Z_2 and U(1) formulations. A qualitative difference occurs inside the vortex core and can be potentially detected by scanning tunneling spectroscopy (STS). We emphasize that here we consider a true vortex with the core situated in the bulk superconductor, as opposed to the flux trapped in the hole. As stated above, the singly quantized Z_2 vortex (binding a vison) is essentially identical to the singly quantized holon vortex in the U(1) theory. However, in the U(1) theory there is a transition with increasing doping to the state in which the spinon vortex becomes energetically favorable [7]. Spectroscopically, holon vortex should exhibit a pseudogap-like local density of states (LDOS), of the type currently observed in experiments on Bi_2-$Sr_2CaCu_2O_8$ [11,12]. Spinon vortex, on the other hand, should exhibit a more conventional LDOS

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[3] The authors are indebted to P.A. Lee for pointing out this fact and for correcting their original misunderstanding of the SF experiment in the SU(2) theory.

with large resonance at the Fermi level, as predicted from d-wave BCS theory [13,14]. Transition from holon to spinon vortex as a function of doping predicted by U(1) theory should therefore be directly observable by STS. In contrast, as argued below, no such transition occurs within the Z_2 theory.

Existence of the spinon vortex in the U(1) theory is predicated upon the fact that the fictitious gauge field **a** has the same symmetry as the electromagnetic field **A**. Therefore, if it becomes energetically favorable, **a** can completely screen the applied magnetic field in the holon term of Eq. (1) and shift the singularity to the spinon term. Z_2 gauge field, being by definition discrete, cannot do this and there can only be one type of a vortex in the Z_2 theory.

In the SU(2) theory the holon condensate kinetic energy has the form [15].

$$|(\nabla + i\mathbf{a}^{(3)}\hat{\tau}_3 - ie\mathbf{A})z|^2, \qquad (4)$$

where $z = (z_1, z_2)$ is the SU(2) holon doublet, and $\mathbf{a}^{(3)}$ is the component of the gauge field associated with the $\hat{\tau}_3$ Pauli matrix. Because of the matrix structure of Eq. (4), gauge field $\mathbf{a}^{(3)}$ can screen magnetic flux seen by one component of z, but *not both*. As discussed by Lee and Wen [4], this results in 2π phase winding and suppression in the core of one of the components of z. Accordingly, the vortex core in SU(2) theory will be in a staggered flux phase. In the SU(2) theory the staggered flux phase is a gauge equivalent of the fermion pairing phase but it is easy to see that $\mathbf{a}^{(3)}$ does *not* couple to the fermions as a magnetic field. There will therefore be no analog of the spinon Meissner effect with respect to $\mathbf{a}^{(3)}$ and the gauge flux can escape from the hole when the superconducting order is supressed.

In summary, we argued that in a spin–charge separated superconductor the general outcome of the SF experiment [2] will be the same for U(1) and Z_2 theories. This is a direct consequence of the fact that charge carrying bosons are assumed to Bose-condense individually and the singly quantized magnetic vortex must therefore bind additional flux quantum of the fictitious gauge field. When such a vortex is trapped in the hole and then removed by heating above T_c, the fictitious flux cannot escape below the pseudogap temperature T^* because it continues to experience the Meissner effect caused by the spinon pair condensate. The trapped flux can then be detected by cooling down below T_c where it necessarily binds a quantum of physical magnetic field.

In the event of positive outcome of the SF experiment it could make sense to carry out detailed spectroscopic study of the vortex cores in cuprates as a function of doping in order to establish the symmetry of the fictitious gauge field. We argued that Z_2 theory can support only one type of a vortex with pseudogap-type spectrum in the core. U(1) theory, on the other hand, predicts a transition from the holon vortex in the underdoped to the spinon vortex in the overdoped region with qualitatively different spectroscopic signatures [7]. In the SU(2) theory of Lee and Wen [10] the SF experiment will not work. Lee and Wen [4] argued for a vortex core in the staggered flux phase, and proposed various probes to detect its signature. Experimentally there exists evidence for one type of a vortex in $Bi_2Sr_2CaCu_2O_8$ with pseudogap-type spectrum [11,12], but detailed studies of strongly underdoped and overdoped regions have not yet been completed.

Acknowledgements

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